1.2 OK, so why learn math?

If you've read this far, you've likely boarded the "we should think of math as a language" train. But this still begs the question: Why learn math? If math is just another language, then why not learn Spanish or Mandarin instead? What we are really asking here is: What is the use of speaking math? And to answer that, we need to answer the question: *What* is math a language *of*?

As you may have guessed, math is not the native language of a certain people, a culture or society that you're born into, where, instead of screaming "Mama," hungry babies scream "addition," although I'm quite enjoying this mental image. So when I say that math is the language of *something*, that *something* is not a people. It is a topic, a line of thinking that the language called math (the symbols, the phrases) was invented to speak about. And this topic is *patterns*.

Let me explain what I mean: as I am sure you are aware, there are certain concepts in spoken languages that are more easily discussed in some languages than in others — presumably because the culture of native speakers is predisposed to finding the concept important and thus adapting the language to suit their needs, although I am no linguistic anthropologist. *Schadenfreude* is a word that comes to mind in this context: it describes the unique pleasure gained when another person is observed to experience ill fate. Usually this doesn't refer to a severe incident, but *Schadenfreude*[4] might be the reason why you giggle when a colleague you're not particularly fond of spills coffee all over their white shirt just before an important meeting. I will not speculate as to why this word had to be invented in a Germanic culture, but I will hypothesize that this concept seemed to not exist in the English language — it was not possible to speak about it unless in an inherently clumsy way — which is why *schadenfreude* is now also an English word.

In the same vein, there are topics that *any* spoken language around the world is just ill-equipped to handle. One such topic is the concept of *patterns*. And, I'm sure it is no surprise to you, observant reader, that math is the language that was created to speak about exactly this topic. The idea of math being the language of patterns will come up repeatedly in this book, it is just that important! And while one example will be woefully insufficient in getting this point across, let me nonetheless give you a glimpse of just how efficient a language math is when it comes to patterns.

But first, we have to answer the question: What actually *is* a pattern? A quick Google search will reveal to you that the word *pattern* refers to something that can be repeated over and over again. I'm sure you are familiar with decorative patterns on, for example, fabrics where a certain motif is repeated over the entire length of the material. But we can also speak about patterns in behavior, that is, trends that keep playing out over and over. The math language is actually perfectly suited to speaking about both types of these patterns! Let me give you

[4] In German, this word is spelled with a capital "S."

an example of the latter:

Have you ever noticed that when you tear a hole into a net you actually end up with *fewer* holes than you had before? Maybe you're even tempted to get a net to try this out; I'm sure after a few tries you'll be inclined to believe me. But now one has to question: Is this trend we're observing always true? Will tearing a hole in a net *always* result in fewer holes than before? Is this a true *pattern*, or are there exceptions?

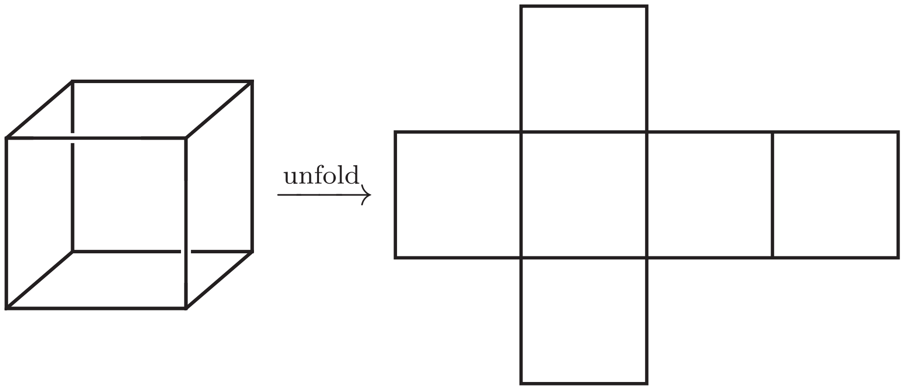
It turns out, this is, in fact, a true pattern, but in English, you would have to go through some clunky reasoning to explain this (go ahead, dear reader, try it!). In math, you may simply say:

$$E - V = F - 2 \tag{1.6a}$$

$$(E \to E - n, V \to V)\ , n \in \mathbb{N} \quad \Longrightarrow \quad F \to F - n\ . \tag{1.6b}$$

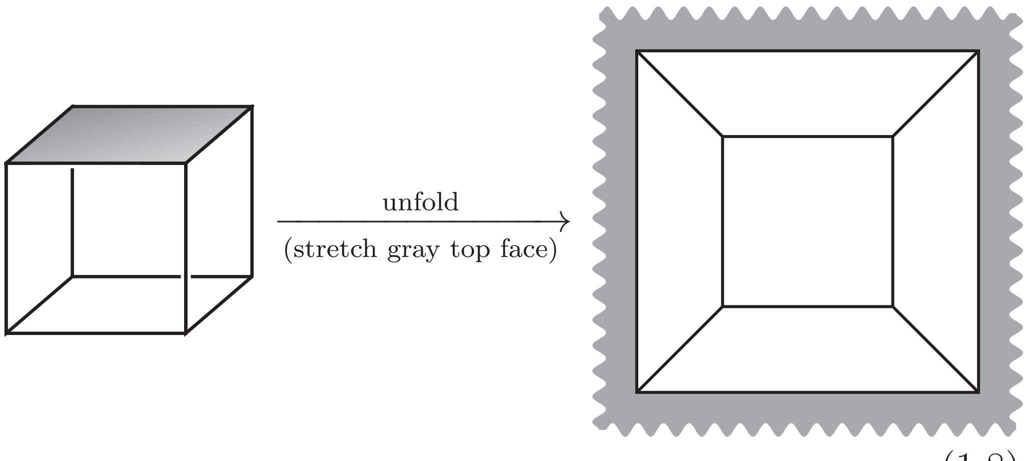
OK, hold your horses, I am aware this is super condensed and uses symbols I have not yet explained, so I don't actually expect you to *understand* what is written in the above two lines! My aim is to illustrate that the mathematical statement is very short and concise. "In math," it is possible to state exactly what is happening to the net that is being torn, while, in English, we would probably still be fumbling about.

Oh, you want to know what Eq. (1.6) is actually saying? All right, I won't go into too much detail (you will learn your first bit of math vocabulary beyond basic arithmetic in the following Section 1.3, and I don't want to overwhelm you), but I will give you an overview. Instead of a net, let's first imagine a polyhedron (you will see why shortly) which is just a solid body consisting of sharp corners, straight edges, and flat faces. For example, a cube and a pyramid are polyhedra (the plural of polyhedron), but a cone is not a polyhedron because it is round, so it doesn't solely consist of flat faces. Now let's try unfolding such a polyhedron. For example, you may have already used a cutout like the following one to fold a cube:

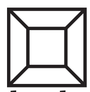


. (1.7)

OK, but now it looks like the unfolded version has more edges than the folded one — when folding, some of the drawn edges will combine into one edge. That's no good: the unfolded version should really have the same number of edges as the folded one. So instead, imagine stretching one face really far out, so much so it tears, and then the whole cube will flop apart (unfold) with the stretched-out face surrounding the rest — this is a little hard to visualize, but I hope the following drawing helps:



(1.8)

What you have now is something that looks like a net,  , where each of the faces of the original cube is a hole, plus the big "hole" on the outside of the net (the stretched out face of the cube). And maybe you can imagine how each net can be made into a polyhedron by reversing this process: imagine pulling the string surrounding the net to close it up, and the little hole that is left is the last face of the polyhedron that was stretched out.

Fine, but why did we go through all of the trouble to think of a net as a polyhedron and vice versa? Well, the answer is owing to a Swiss mathematician from the 18th century by the name of Leonhard Euler (remember this name, dear reader, for there is more to come from good Leonhard later in this book). From his works, I imagine Euler to be quite the playful fellow, as a child always tinkering about and asking "but why??" to the vexation of his parents. At some point, Euler must have played around with a whole lot of polyhedra, because he noticed (and proved!) a curious *pattern* in the relationship between the number of sharp corners, straight edges, and flat faces of any polyhedron [2]: namely, if you subtract the number of corners from the number of edges, you will always get 2 less than the number of faces. This pattern is what the top line in Eq. (1.6) describes, where E denotes the number of *e*dges, V is the number of corners (or *v*ertices), and F is the number of *f*aces. Don't believe me? Try it out for the cube, where you have 8 corners (so $V = 8$), 12 edges ($E = 12$), and 6 faces ($F = 6$): $12 - 8 = 6 - 2$.

Since we just argued that a net is basically just a polyhedron unfolded in a strange way, everything that Euler discovered holds true for polyhedra must also be true for nets! In particular, nets must obey Euler's relationship, where F now denotes the number of holes of the net (not forgetting the big "hole" on the outside), E is the number of strings between knots, and V is the number of knots. And so, if we change the number of strings (edges) by tearing the net, the number of holes must also change according to Euler's formula, which is exactly what the bottom line in Eq. (1.6) says. There we go, that's it!

In summary, Euler discovered a pattern in the relationship between flat faces, straight edges, and sharp corners of any polyhedron, which, as a consequence, allows us to observe the pattern that a net always has fewer holes when torn. And these patterns are neatly described in Eq. (1.6).

I cannot help but point out the irony that even just this super condensed explanation of Eq. (1.6) in words and pictures took several somewhat lengthy paragraphs, while the equation itself is incredibly short. This once again exemplifies my point that math, as a language, is uniquely suited to talking about patterns — it has the right vocabulary and the right grammar to speak about them. However, it is only understandable to those who *speak* math.

And with this, we have arrived at the central message of this chapter: if we think of math as a language rather than an innate talent, we put into perspective our own expectation and sense of capability. If you don't speak Russian, you wouldn't beat yourself up for not understanding Tolstoy's *War and Peace* in the original language. You would not think of yourself as stupid or incapable for not being able to decipher it; you would simply recognize that understanding Russian is not a skill that you currently possess but one you could definitely learn. And, even more so, if you started learning Russian, even if you'd been at it for quite some months and still couldn't read *War and Peace* in the original, you would understand that your language skill level is not quite on par with the piece. You would understand that Tolstoy was a great writer who used language at an advanced skill level, which you, a novice learner, have just not obtained...*yet*. Let's start putting math into perspective: if you don't understand a certain formula or line of reasoning, steer clear of thinking that this is a reflection of your innate capabilities to "do math," and rather recognize that your "math language skills" are simply not yet at the level required by the formula in question. With practice and dedication, you can get there!

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